θ VACUUM IN A RANDOM MATRIX MODEL

ROMUALD A. JANIK[1,2], MACIEJ A. NOWAK[2,3], GÁBOR PAPP[4]
and ISMAIL ZAHED[5]

[1] *Service de Physique Théorique, CEA Saclay, F-91191 Gif-Sur-Yvette, France.*
[2] *Department of Physics, Jagellonian University, 30-059 Krakow, Poland.*
[3] *GSI, Planckstr. 1, D-64291 Darmstadt, Germany*
[4] *CNR Department of Physics, KSU, Kent, Ohio 44242, USA &*
HAS Research Group for Theoretical Physics, Eötvös University, Budapest,
[5] *Department of Physics and Astronomy, SUNY, Stony Brook, New York 11794.*

Inspired by recent lattice calculations, we model certain aspects of the θ-vacuum using a matrix model with gaussian weights. The vacuum energy exhibits a cusp at $\theta < \pi$ that is sensitive to both the accuracy of the numerical analysis and the maximum density of winding modes present in a finite volume.

Recent lattice study of the CP^3 model on the lattice[1] have revealed an intriguing result: the energy as a function of the CP breaking θ angle levels off at large θ. The result is very interesting, since in that model the string tension for small charges is related to the θ-derivative of the vacuum energy. A plateau indicates the loss of confinement. The result was reexamined in [2] where the authors pointed out to some subtleties regarding the technical methods applied.

In this talk we will address these issues using a random matrix model with the partition function

$$Z(\theta, N_f) = \left\langle \prod_{j=1}^{N_f} \det \begin{pmatrix} im_j e^{i\theta/N_f} & W \\ W^\dagger & im_j e^{-i\theta/N_f} \end{pmatrix} \right\rangle . \tag{1}$$

where the averaging is carried over the matrices W, $W^\dagger$ using the weight $\exp\left(-n/2\mathrm{Tr}W^\dagger W\right)$ and over the matrix size n and σ, with weights $\exp\left(-\chi^2/2\chi_* V\right)$ and $\exp\left(-\sigma^2/2\sigma_* V\right)$, respectively. Here W is a complex asymmetric $n_+ \times n_-$ matrix, $n = n_+ + n_-$, and $\sigma \pm \chi = 2n_\pm - \langle n \rangle$. The mean number of zero modes $\langle n \rangle$ is either fixed from the outside or evaluated using the gaussian measure. Here and for simplicity we use the quenched measure without the fermion determinant to fix $\langle n \rangle$. Throughout, we set the value of the quark condensate to $\Sigma = 1$ in the chiral limit.

Eq. (1) is borrowed from the effective instanton vacuum analysis [3] where n_+ counts the number of right-handed zero modes , and n_- the number of left-handed zero modes. The number of exact topological zero modes is commen-

surate with the net winding number carried by the instantons and antiinstantons. By analogy with [3], χ_* and σ_* will refer to the unquenched topological susceptibility and particle compressibility, respectively, with $\sigma_*^2 = 12\mathbf{n}_*/11N_c$ and $\mathbf{n}_* = \langle n\rangle/V$ the mean density of zero modes. If the compressibility σ_* is assumed small in units of $\Sigma = 1$, then typically $n \sim \langle n\rangle$.

quenched case: In the case of the quenched partition function ($N_f = 0$) one probes the nature of the gaussian measure. We start discussing the case where $\langle n\rangle = \infty$ with no restriction on the value of n, and hence no restriction on the value of χ. Then, we discuss the case where $\langle n\rangle$ is large but finite, so that $|\chi| \leq n$ with typically $n \sim \langle n\rangle$ for a peaked distribution in n.

When the sum is unrestricted and infinite, using Poisson resummation formula we have

$$Z_Q(\theta) = \sum_{k=-\infty}^{+\infty} e^{-\frac{1}{2}V\chi_*(\theta-2\pi k)^2} = \theta_3(\theta/2, e^{-\tau})\,. \tag{2}$$

with θ being the third elliptic function and $\tau = 1/(2V\chi_*)$. The result is manifestly 2π periodic. The vacuum energy, $F_Q(\theta) = -\ln Z_Q(\theta)/V$ as $V\to\infty$ is simply $F_Q(\theta) = \min\frac{1}{2}\chi_*\left(\theta + \mathrm{mod}\,2\pi\right)^2$ in agreement with the saddle-point approximation. This result is in agreement with the result using large N_c arguments [4], and recent duality arguments [5]. We observe that the cusp at $\theta = \pi$ (mod 2π) sets in for $V = \infty$.

In the matrix model being considered the sum over χ is restricted to $|\chi| < N$, with $N = \max n$. We denote by $\mathbf{n} = N/V$ the maximum density of winding modes. While in general $\mathbf{n} \neq \mathbf{n}_*$, for a peaked distribution in n (small compressibility σ_*) we expect $\mathbf{n} \sim \mathbf{n}_*$. This will be assumed throughout unless indicated otherwise. Hence

$$Z_Q(\theta) = \sum_{\chi=-(N-1)}^{N-1} e^{i\theta\chi} e^{-\chi^2/2V\chi_*} \tag{3}$$

Approximating the sum in (3) by an integral and evaluating it by saddle point we obtain $Z_Q \sim e^{-V\chi_*\theta^2/2}$ apparently in agreement with the infinite sum. However, the Euler-MacLaurin summation formula shows that deviation from this result in the case of a finite sum is expected for $\theta > \theta_c \sim \mathbf{n}/\chi_*$ [6]. Indeed, this is confirmed by detailed numerical calculations of the vacuum energy as shown in Fig 1 (left). For $N = 250$ and $\chi_* = 1$, the double precision (16 digit) numerics (circles) breaks away from the saddle point approximation (solid line) at $\theta/\pi \sim 0.2$ for both $\mathbf{n} = 1$ and $\mathbf{n} = 4$, while the high-precision (64 digits) calculations (dashed line) agree with the saddle point result at

$\mathbf{n} = 4$ but break away at $\theta/\pi \sim 0.3 \approx 1/\pi\chi_*$ at $\mathbf{n} = 1$. On the right we show the numerical result for the CP^3 model [1] showing a similar behavior.

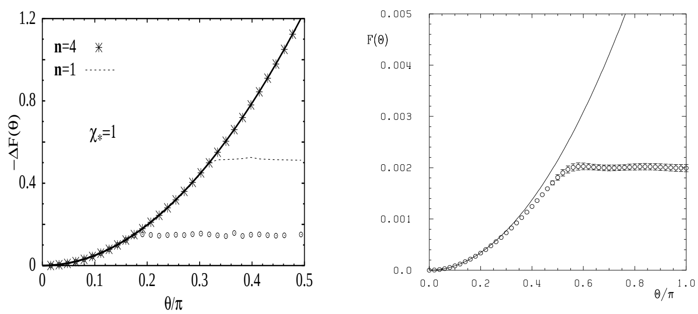

Figure 1. Free energy in a quenched model (left) and in the CP^3 model (right). See text.

unquenched case: Analytical and numerical studies of the unquenched matrix model for $N_f = 1$ and $N_f = 2$ [6] lead to results similar to the ones observed in the quenched case. The mean-field result breaks down for $\theta > \theta_c \sim \mathbf{n}/\chi_{top}$. For small quark masses $\chi_{top} \sim m$ [7], the breakdown is observed only for ensembles with very small maximal winding number.

In this talk we have shown that the vacuum energy of a chiral matrix model developes a cusp at $\theta < \pi$ that is sensitive to the numerical accuracy. The cusp persits at high accuracy if the maximum winding number considered is low, in disagreement with mean-field results. Similar observations in current lattice simulations should therefore be interpreted with care.

Acknowledgments

This work was supported by the US DOE grants DE-FG-88ER40388, by the Polish Government Project (KBN) grant 2P03B00814 and by the Hungarian grant OTKA-F026622.

References

1. G. Schierholz, *Nucl. Phys. Proc. Suppl.* **A37**, 1994 (203).
2. J. C. Plefka and S. Samuel, *Phys. Rev.* D **56**, 1977 (44).
3. M. Nowak, J. Verbaarschot and I. Zahed, *Phys. Lett.* B **228**, 1989 (251).
4. E. Witten, *Annals of Phys.* **128**, 1980 (363).
5. E. Witten, *Phys. Rev. Lett.* **81**, 1998 (2862).
6. R.A. Janik, M.A. Nowak, G. Papp and I. Zahed, hep-ph/9901390.
7. R.A. Janik, M.A. Nowak, G. Papp and I. Zahed, *Nucl. Phys.* B **498**, 1997 (313).